Evaluation of Software Development Investments: a Real Options Approach

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Abstract. Software development projects are exposed to multiple sources of uncertainty. This uncertainty has an enormous impact in the investment economic value. The variability of a software development project payoff can be represented by the volatility of the project value over the analyzed period. The aim of this work is to define a framework to estimate the volatility of IT investments that takes into account all relevant information that has impact on project value, and show how to use this volatility estimation in a real options analysis. The suggested method could help IT managers produce a well-structured valuation process in software development investment decision-making, and understand the interactions between software process, market environment, financial issues and options value in a clear way.

Keywords: Software economics, Real Options Analysis, Investment Analysis, Risk Management, Software Management, System Dynamics

1 Introduction

Just as investors address their objectives for risk and return using portfolios of financial investments, firms use Information Technology (IT) portfolio management to better enable their management teams to mach IT investments to their strategic objectives [1]. The IT portfolio encompasses total IT spending in the enterprise from operating expenses, technology, services, digitalized information, outsourcing, and people dedicated to IT. Evaluating and justifying IT investments can pose problems different from traditional capital investment decisions. Organizations often use net present value (NPV) calculations for cost-benefit analyses. In an NPV analysis, analysts convert future values of benefits to their present-value equivalent by discounting them at the organization´s cost of funds. They then can compare the present value of the future benefits to the cost required to achieve those benefits, in order to determine whether the benefits exceed the costs [2]. But NPV analysis works well in situations where costs and benefits are well defined and can easily be converted in monetary values. The value of IT projects depends on company´s internal operations, changes in process, technology, people, organization and culture. Weill (2009) found that executives have four different management objectives for investing in IT: transactional, informational, strategic and infrastructure. The fact that organizations use IT for different purposes further complicates the costing process. In addition, although IT projects are an important mechanism for delivering value from the IT function they are prone to failure. The Standish Group reported that 35% of IT projects are successful, 19% fail, and projects are estimated to have an average cost overrun of approximately 54% [3]. These numbers are difficult to ignore and suggest that care should be taken when evaluating projects performance and costs.

The problem of evaluating investments in IT projects has been extensively addressed in the literature. Software cost estimation techniques focus on predicting the amount of effort required to build a software system. Approaches such as Boehm´s constructive cost model (COCOMO) [4] or Putnam´s software life cycle management [5] rely on mathematical formulas and use of software characteristics (such as software size or software reliability) to predict software life cycle costs and project schedules. On the other hand, financial literature considers financial business ratios (return on investment, payback period, net present value) and cost oriented approaches (zero base budgeting approach, cost effectiveness analysis).

Recently, real option theory applies financial option theory to IT investments and aims at quantifying the value of management flexibility in a world of uncertainty (*e.g.* unexpected market development). A real option is the right, but not the obligation, to undertake some business decisions. Research on real options is mainly concerned with the identification of various options in IT investments, and then framing as pricing problems, their valuation, and interpretation of results [6]. Hence, real option theory goes beyond traditional financial business measures which do not allow capturing the value of IT investments in environments of change. Under the binomial model, five parameters are needed to determine the option price. These are the current stock price, the strike price, the time to expiration, the volatility of the stock price, and the free-risk interest rate. The volatility of the stock price is a statistical measure of the stock price fluctuation over a specific period of time. Real options approaches are generally based on the assumption that financial markets provide valuable information sources to assess the market uncertainties. A common solution is to find a publicly owned firm operating in the same market, which is assumed to be subject to the same market risks [7]. However, as mentioned earlier, volatility in IT projects context are based on company´s internal operations, are managed by changes in process, technology, people, organization and culture. Hence, company-specific risks measures should be used to determine volatility. The aim of this work is to define a framework to estimate the volatility of software development investments that takes into account all relevant information that has impact on project value, and show how to use this volatility estimation in a real options analysis. We develop a simulator that can be used to better understand process dynamics of system engineering and evaluate investments. Such a tool would allow answering which factors impact project volatility and how to quantify them.

The remainder of this paper is organized as follows. Section 2 describes the risks associated with IT investment, and gives an overview of software process modeling based on system dynamics. Section 3 gives a brief introduction of the binomial option pricing model. Section 4 presents an approach which includes volatility estimation based on system dynamics modeling of software projects and real options analysis. Section 5 applies the methodology to an example. Finally, strengths and limitations of the proposal are discussed.

2 Software Process Modeling based on System Dynamics

Modeling based on system dynamics was developed by Jay W. Forrester and has become relevant in the last years since the need to model complex systems. System dynamics is a methodology for modeling the forces of change in any dynamically complex system so that their influences can be better understood. The methodology is iterative, and allows various stakeholders to combine their knowledge about a problem in a dynamic hypothesis and then, using computer simulation, to formally compare many scenarios about how to introduce change [8]. The emphasis is not in predicting the future but in learning how actions in the present can trigger reactions in the future [9]. Even when it is not possible to define with a certain degree of confidence constant values o ratios of change, the model is used as a learning tool to determine causal relationships and relevant factors.

The first work to apply it to software engineering is Tarek Abdel-Hamid dissertation on Software Project Dynamics [10]. They develop a core integrated system dynamics model for software development project management. Since then many research has been done in the area. The scope of a software process simulation is generally a portion of the life cycle, a development project, multiple concurrent projects, long-term product evolution, long-term organization [11]. Typical result variables for software process simulation include effort, cycle time, and defect levels, staffing requirements over time, return on investment, throughput, and productivity. In [12] the authors present an experimental investigation about staffing delays in software project management. Madachy developed many experimental models and others that have been used by industry [13]. Ferreira illustrates a software business model that considers the effects of requirements volatility on a software project´s key management parameters such as cost, schedule and quality. The authors administered a survey to collect information for a subset of the factors identified in the causal model and quantify the level of relationships [14]. Recently, in [15] and [16] the authors propose a model to analyze the causes of accidents based on system dynamics. The model known as STAMP (System Theoretic Accident Model and Processes) considers systems as interrelated components. Systems are not treated as a static design but as a dynamic process that is continuously adapting to fulfill its objectives and react to changes of the system and the environment. In this way, it is possible to model the influence of issues such as budget cuts, complacency or schedule pressures. In particular, once relevant risks have been identified, system dynamic models are used to analyze the impact on different parameters of the security program; analyze different modes of operation; and identify measures that point an increment in risks.

3 Binomial Option Pricing Model

The binomial options pricing model provides a general numerical method for the valuation of options. The binomial model was first proposed by Cox, Ross and Rubinstein [17]. The model is based on the description of an underlying instrument over a period of time rather than a single point. The original method proposed by Cox, Ross and Rubinstein can be briefly described in three steps: generation of the binomial price tree; calculation of option value at each final node; and backwards calculation of the option value.

The tree prices are produced by working forward from valuation date to expiration [18]. At each step, it is assumed that the value of the option will move up or down by a factor. The up and down factors are calculated using the underlying volatility and the time duration of a step. Starting from an initial expected value, V moves either up to uV with probability p or down to dV with probability $1-p$, in a fixed interval Δt, where $u>1$, $d<1$, and $p=(e^{r_f t}-d)/(u-d)$ with r_f being the risk free rate corresponding to the life of the option. The process may be repeated for multiple periods. When the volatility is σ, then u and d can be determined as $u=e^{\sigma\sqrt{\Delta t}}$ and $d=1/u$.

Then a decision tree could be established to determine the real options value underlying the investment. The option value is found at each node, starting at final nodes and working back to the first node of the tree. The binomial value is found recursively at each node. For the valuation of strategic flexibility contained in the investment decisions we apply the approach of [17]. The value of the V_{ij} option at each nodes is:

$$V(i,j)=e^{-r_f\Delta t}\left[pV_{(i+1,j)}+(1-p)V_{(i,j+1)}\right] \quad (1)$$

where i is the number of upward movements and j is the number of downward movements at step $(i+j)$.

If exercise is permitted at the node, then the model takes the greater of binomial and exercise value at the node. Let I be the option´s exercise price, CF_{ij} the cash flow of node i,j, then the value of a call option on *V* that matures in Δt is:

$$max\{0, VAN(CF_{ij})-I\} \quad (2)$$

In this paper we use a lattice model based on a recombining tree. A binomial tree is recombining when for any tree node all paths that lead to the node contain the same number of “upward moves” and “downward moves”. Thus the key feature of the lattice model is that an up move followed by a down move leads to the same value as a down followed by an up. While an *n* period recombining binomial lattice has a total of $(n+1)(n+2)/2$ nodes, an equivalent binomial tree has $2^{n+1}-1$ nodes [19].

4 The Proposed Valuation Framework

All processes must be defined, implemented, deployed, monitored, and continuously adapted to changing requirements and conditions. Software process modeling based on System Dynamics provides a good foundation to represent a software development project and use it as a project management tool that aids in monitoring metrics such as effort, time delays and costs. Ideally, the model should be as complete as possible including issues that may have impact on the value of the project (e.g. market behavior, internal operations, culture). Hence, computer simulations based on the model allow inferring which variables have more impact on the net present value (NPV) and quantify the impact. This information is gathered to estimate the volatility of the NPV and perform a real options analysis of the project. The proposed working framework is divided in 3 steps as described below.

Step 1: Modeling Software Process, Marketing and Financial Systems. In order to include a comprehensive model and at the same time keep this presentation as simple as possible, we have adapted a model based on Value based Software Engineering (VBSE) [20] developed by Madachy [21]. VBSE seeks to integrate value considerations into current and emerging software engineering principles and practices [22].

The model includes three subsystems (see Fig. 1): Software Process and Product, Market and Sales, and Finances. Software Process and Products provides an estimation of effort and product quality. Quality has an impact on Market and Sales subsystem. Finally, effort and sales feed the financial subsystem to calculate the NPV of the project. The Software Process Model considers that the introduction of errors causes an increase in the effort and can increase the length of the project. The Market and Sales Model assumes that when the market perceives an increase in quality sales increase. However, quality perception is not instantaneous.

Step 2: Conduct Sensitivity Analyses and Estimate Project Value Volatility. A sensitivity analysis allows identifying the variables that have impact on NPV. Finally, we define simulation experiments where relevant variables are perturbed. As a result, the sensitivity analyses generate a NPV distribution reflecting the induced variation of the impact factors. The estimated standard deviation quantifies the volatility of NPV that arises from uncertainty in input variables.

Step 3: Structure the Project as a Real Options Problem. In the last step, we can assess the real option value of the investment based on the results obtained above. We follow the traditional three-step process: generation of the binomial price lattice; calculation of option value at each final node; and backwards calculation of the option value (see Section 3). We do not use European options since they cannot be exercised until the day of expiration.

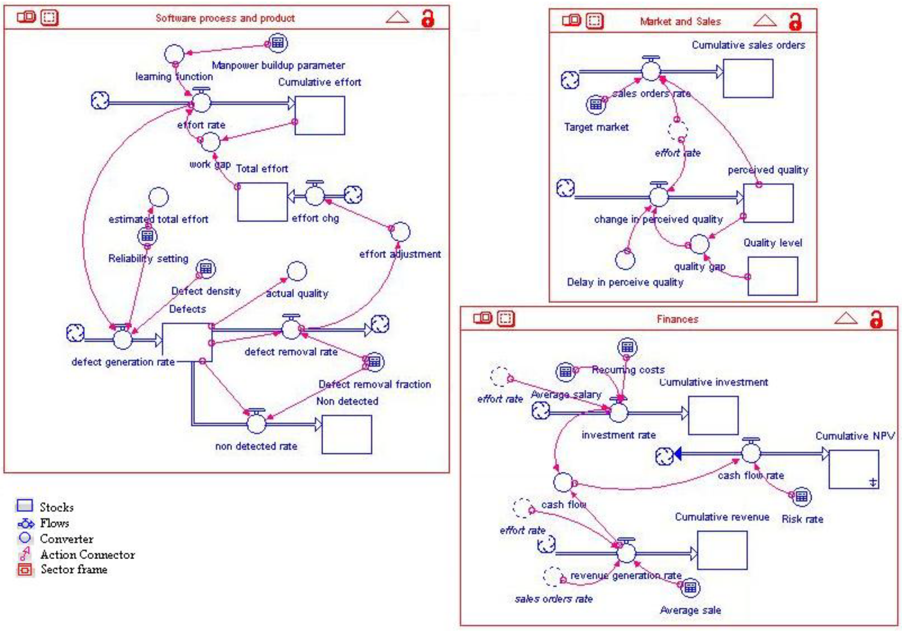

Fig. 1: Stella™ implementation of VBSE model

5 Example

In order to illustrate the application of the approach consider the following example adapted and extended from [23].

Background. WayOut Widgets, Inc. has made several incremental upgrades to its web site. Following each upgrade, they have experienced performance problems resulting in numerous complaints, lost sales, and increased demand for human operators to take orders over telephone as customers abandon the web site. Fixing these problems has required hardware upgrades and post-deployment refactoring efforts to tune the software. Refactoring efforts have involved the entire development team for periods ranging from 3 to 12 months. Hardware upgrades have required additional application and database servers.

A group of developers has proposed using Software Perfomance Engineering (SPE), a quantitative approach to constructing software systems that meet performance objectives. The SPE initiative will be introduced for the development of the next release of the Web application. This project will involve 15 developers and is expected to take 15 months. The cost worksheet is included in Table 1. The benefits of SPE in this project arise from avoiding costs due to poor performance. For the upcoming project, the estimated amount of time that would be required for refactoring if SPE is not used is 6.5 months. The marketing estimate is that 100 sales per day were lost due to customers abandoning the Web site due to poor performance. These losses would occur every day during the expected 6.5 months refactoring period. The average sale is $50. After deploying each previous release, the company needed to hire 10 agents to handle the increased telephone order volume due to customers abandoning the web site. The cost of the agents for the expected refactoring time is $325.000. Additionally, refactoring would require a server upgrade ($600.000), 15 developers for 6.5 months ($812.500).

Table 1: Costs of SPE Project

Initial costs		Annual costs	
Tools		Software maintenance	12.100
Perfomance modeling tool	8.000	Salaries	
Load driver	70.000	Perfomance analyst	100.000
Workstation	4.000	Continuing education	2.200
Training		**Total annual costs**	**114.300**
In-house training	66.846		
Perfomance Engineer	5.923		
Consulting/mentoring	250.000		
Total initial costs	**404.769**		

So this situation can be structured as an investment problem consisting of three mutual exclusive alternatives:

1. To develop an upgrade using SPE. The initial investment is of $404.769; the monthly operative costs ascend to $9.525. Before the upgrade is launched the rate of increment in sales is estimated in 8%.
2. To develop an upgrade without SPE. Post-deployment refactoring efforts to tune the software require a spending of $1.737.500; there are not incremental operative costs and the system will be ready in three months. The rate of increment in sales before refactoring is estimated in 3% per month.
3. Do not develop an upgrade.

The first comparison is calculating the expected NPV of the project without options using a deterministic discounted cash flow analysis based on a risk-adjusted discount rate (see Table 2). So far it looks like the first and second alternatives should be discarded. The NPV of the first alternative ($1.126.567,28) is smaller than the NPV of the third. The NPV of the second alternative ($266.584,36) is much smaller than the initial inversion ($1.737.500,00). Additionally, the value of this alternative is limited because of its non-strategic role.

Table 2: Net Present Value Analysis

	SPE	Upgrade with refactoring	Do not upgrade
Initial Inversion	404.769,00	1.737.500,00	--
NPV	1.126.567,28	266.584,36	1.582.986,80

In what follows we provide a more realistic estimation of the NPV using a model that integrates the software development process, the marketing environment and financial behavior. We model the software process, marketing and sales of the SPE alternative since it is the options that introduce uncertainty. The VBSE model was implemented in Stella™ [24] and simulations were performed using a time step of 0.1 and Runge-Kutta integration method of fourth order.

The effort rates follows a Norden learning model. When the cumulative effort (measured in month per month) reaches the total estimated effort, the development period finishes. As the effort rate increases, it is assumed that more errors are introduced and quality increases. Then, the effort rate is adjusted to account for the effort required to remove errors (see Fig. 2).

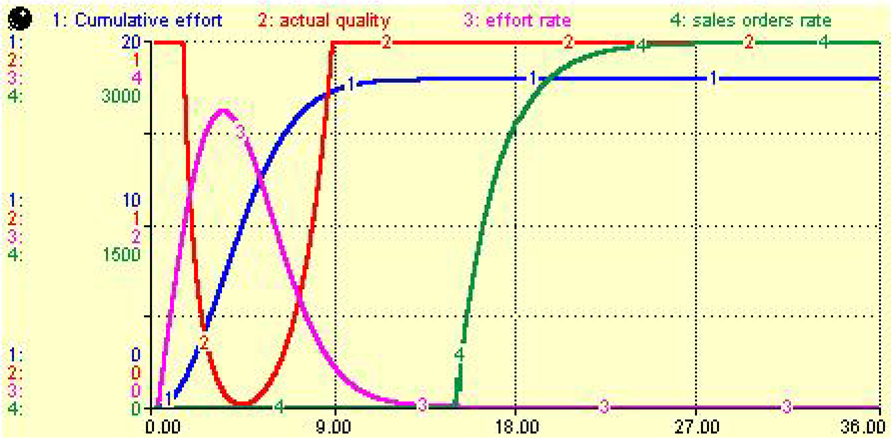

Fig. 2: Development period (15 months) followed by sales

One of the sources of greatest uncertainty is the productivity of developers, which can increase the generation of errors, require more developers, and extend of the duration of the project. Sensitivity analysis shows that *Manpower buildup parameter*, the *defect removal rate* and the *defect density* may extend the length of the project. For example, when the *Manpower buildup parameter* assumes the values 0,1; 0,15; or 0,2, the development takes 15, 13, and 11 months; and the Cumulative Net Present Value is 11.554.654,18; 11.191.345,73; or 10.498.436,25 at the end of simulation period (month 36).

Based on the results of a simulation experiment including 100 replicas and assuming a normal distribution with mean 1.15 and standard deviation of 0.05 for *Manpower buildup parameter*, the Cumulative NPV can be described with a normal distribution with mean 10.994.299,20 and standard deviation of 937.482,64. The estimated standard deviation quantifies the volatility of NPV (8,48%).

We assume a geometric brownian model for the underlying stochastic process (of the cash flows); the time step is $\Delta t = 1$. The up and down factors $u = e^{\sigma\sqrt{\Delta t}}$ and $d = 1/u$ are of 1,088 and 0,918. The stochastic process of SPE´s NPV could be structured as a binomial lattice as shown in Fig. 3 (Appendix). At each point in the lattice the value may go up with probability $p = 0{,}5028$ or down with probability $(1 - p) = 0{,}4971$.The simulated duration of the development was 15 months and by the 18th month sales reached the target market of 5000 sales per month. A decision lattice could be established to determine the real options value underlying the investment (see Appendix: Fig. 4). The decision whether to undertake the upgrade would be made in the initial period depending on whether the payoff is positive. The root value of the decision lattice represents the option value of the investment: $17.335,49 represents the payoff generated by the strategic flexibility. Based on this analysis, management would be inclined to take the SPE project. The use of SPE involves high initial costs that are expected to be recovered from the cash inflows of the following months. This is due to the fact that the staff need to be trained and some time may pass before they are productive enough. Once the upgrade is implemented, sales would increment as a result of customer satisfaction with the web site performance.

6 Conclusions and Future Research

The present paper develops a real options based approach to evaluate software development investments that are subject to multiple sources of uncertainties. The main contribution of the work is the estimation of the volatility of the investment. By modeling the interactions between the software process, the marketing environment and financial behavior, this approach is able to capture the volatility of the investment based on the behavior of the project. Traditional NPV analysis suggests the SPE is not good enough. The “do not upgrade” alternative has the highest NPV, but this is mainly due to NPV limitations. NPV makes implicit assumptions that management cannot react to deviations from the expected scenario of cash flows. Management´s flexibility to defer, contract, expand or abandon its operating strategy, adds value to the NPV [25]. Real Options Analysis gives a solution different from that provided with the NPV calculation. The value of the investment option has a positive payoff even when we considered a finite time period of 18 months. This result is consistent with the intuition that investing in improving process in the long term might have a better payoff. While postponing the upgrade investment may appear logical at first, holding on to options for too long may result in undesirable consequences such as the loss of market share to competitors. The system dynamics model captures this behavior (increasing sales volume after successful web site upload) and hence computes positive cash inflows.

The strengths of the proposed framework are twofold. First, the evaluation method we present gives an estimation of the volatility of the investment based on current information of the software development and marketing environment. To the best of our knowledge, financial literature suggests using a value based on a similar project (and it is quite difficult to compare software developments). Second, the project is structured as a real options problem. Real options analysis is proved to be a suitable tool to valuate investment under uncertainties [7] [6] [26] [27].

The limitations of the proposed software development investment framework are that the simulation model is dependent on the type of lifecycle model used during development; and that quite a lot of experimental data is needed to populate and suit models to the organizations characteristics. In order to overcome these limitations it is necessary to improve the reusability of the models. In addition, more opportunities to better integrate the framework with existing organizational experience data bases would help in calibrating the simulation model.

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Appendix

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1126567.28	1226267.75	1334791.64	1452919.83	1581502.28	1721464.20	1873812.65	2039643.84	2220150.99	2416632.89	2630503.32	2863301.14	3116701.42	3392527.45	3692763.91	4019571.10	4375300.51	4762511.74	5183990.91
	1034972.86	1126567.28	1226267.75	1334791.64	1452919.83	1581502.28	1721464.20	1873812.65	2039643.84	2220150.99	2416632.89	2630503.32	2863301.14	3116701.42	3392527.45	3692763.91	4019571.10	4375300.51
		950825.43	1034972.86	1126567.28	1226267.75	1334791.64	1452919.83	1581502.28	1721464.20	1873812.65	2039643.84	2220150.99	2416632.89	2630503.32	2863301.14	3116701.42	3392527.45	3692763.91
			873519.53	950825.43	1034972.86	1126567.28	1226267.75	1334791.64	1452919.83	1581502.28	1721464.20	1873812.65	2039643.84	2220150.99	2416632.89	2630503.32	2863301.14	3116701.42
				802498.90	873519.53	950825.43	1034972.86	1126567.28	1226267.75	1334791.64	1452919.83	1581502.28	1721464.20	1873812.65	2039643.84	2220150.99	2416632.89	2630503.32
					737252.53	802498.90	873519.53	950825.43	1034972.86	1126567.28	1226267.75	1334791.64	1452919.83	1581502.28	1721464.20	1873812.65	2039643.84	2220150.99
						677310.96	737252.53	802498.90	873519.53	950825.43	1034972.86	1126567.28	1226267.75	1334791.64	1452919.83	1581502.28	1721464.20	1873812.65
							622242.87	677310.96	737252.53	802498.90	873519.53	950825.43	1034972.86	1126567.28	1226267.75	1334791.64	1452919.83	1581502.28
								571652.04	622242.87	677310.96	737252.53	802498.90	873519.53	950825.43	1034972.86	1126567.28	1226267.75	1334791.64
									525174.45	571652.04	622242.87	677310.96	737252.53	802498.90	873519.53	950825.43	1034972.86	1126567.28
										482475.66	525174.45	571652.04	622242.87	677310.96	737252.53	802498.90	873519.53	950825.43
											443248.47	482475.66	525174.45	571652.04	622242.87	677310.96	737252.53	802498.90
												407210.59	443248.47	482475.66	525174.45	571652.04	622242.87	677310.96
													374102.75	407210.59	443248.47	482475.66	525174.45	571652.04
														343686.70	374102.75	407210.59	443248.47	482475.66
															315743.60	343686.70	374102.75	407210.59
																290072.38	315743.60	343686.70
																	266488.34	290072.38
																		244821.77

Fig. 3: Binomial price lattice (SPE option).

17335.49	26910.12	41263.42	62432.15	93093.25	136620.85	197050.79	278892.09	386729.61	524601.96	695234.21	899361.04	1135545.46	1400934.96	1693040.87	2011684.32	2359216.66	2738197.36	3151412.40
	7793.26	12612.96	20190.41	31931.58	49830.20	76618.78	115887.69	172106.26	250449.54	356317.92	494472.11	667849.40	876432.83	1116978.38	1384640.67	1676680.07	1995256.72	2342722.00
		2982.20	5052.05	8480.19	14089.62	23143.14	37528.05	59975.10	94276.51	145421.91	219503.04	323164.07	462354.64	640321.69	855414.36	1100617.57	1368213.08	1660185.41
			913.07	1626.03	2875.97	5047.82	8783.07	15131.38	25772.25	43317.87	71684.14	116453.98	185028.66	286140.40	428016.87	614419.48	838986.77	1084122.91
				199.42	375.08	702.79	1311.13	2433.97	4492.55	8236.74	14981.54	26988.77	48048.02	84275.07	144984.22	242988.22	392318.52	597924.82
	max(0,17335.49)				23.38	46.69	93.23	186.17	371.73	742.26	1482.11	2959.43	5909.29	11799.46	23560.74	47045.23	93938.23	187572.48
						0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
							0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
								0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
									0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
										0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
											0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
												0.00	0.00	0.00	0.00	0.00	0.00	0.00
													0.00	0.00	0.00	0.00	0.00	0.00
														0.00	0.00	0.00	0.00	0.00
															0.00	0.00	0.00	0.00
																0.00	0.00	0.00
																	0.00	0.00
																		0.00

Fig. 4: SPE investment decision lattice